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Transition Probabilities in the CuCl²⁻-Complex

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Transition probabilities in the CuCl²⁻-complex have been calculated using the L.C.A.O.functions obtained in the preceding paper by P . Ros and G , C , A , S CHUIT. For all transitions the calculated absolute oscillator strenghts turn out to be a factor 2 too large with respect to the experimental values. The calculated transition probabilities are mainly determined by the amount of mixing of the ligand functions in the molecular orbitals. If this mixing is assumed somewhat less the agreement is still better.

Unter Benutzung der L.C.A.O.-Funktionen aus der vorstehenden Arbeit von P. Ros und G. C. A. SCHUIT wurden die Übergangswahrscheinlichkeiten im CuCl²--Komplex berechnet. Die absolute Oszillatorstärke, die hauptsächlich vom Gewicht der Ligandenfunktionen abhängt, wird für alle Übergänge um den Faktor 2 zu groß. Verringert man den Beitrag dieser Funktionen, wird die IJbereinstimmung mit dem Experiment besser.

A l'aide des fonctions L.C.A.O. du travail précédent de P. Ros et G. C. A. SCHUIT, nous calculons les moments de transition pour l'ion complexe $CuCl₄²⁻$. Pour toutes les bandes les forces oscillatrices calculées sont deux fois trop grandes devant l'expérience. Elles sont déterminées surtout par le poids des orbitales des ligandes dans les O.M.; l'accord s'améliore par une diminution de ee poids.

Introduction

In the last twenty years many attempts have been made to calculate the stability and the charge distribution of inorganic complexes. In most of these calculations, more or less empirical, the results were checked by comparing them with data obtained from the optical spectra *[2, 3, 4, 6, 7, 9, 10, 14, 15, 20, 21].* In the present report a calculation of the intensities of the bands in the CuCl²⁻⁻spectrum is described. It is carried out making use of the one electron wave functions and charge distribution arising from a quantitative molecular orbital calculation in which as few as possible empirical parameters are introduced (see preceding paper by P. Ros and G. C. A. SCHUIT and $[19]$. By comparing the calculated intensities with the experimental values the accuracy of the M.O. calculation is checked. By evaluating some terms which up to now have always been approximated or neglected we hope to be able to test and maybe to simplify the usual method of calculating the intensities.

Transition Probability

The probabilities of electric dipole transitions and therefore the intensities of the corresponding absorption bands can be expressed in the oscillator strengths f .

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It is possible to determine / from the optical spectrum, as well as to relate it to the wave functions of the states between which the transition occurs [3, 5, *17].*

t. Theoretical Calculation o/f: The formula for the theoretical calculation of / is given by:

$$
f = 1.085 \times 10^{11} \bar{\nu} P ,
$$

\n
$$
P = A v_{\rm I} \Sigma_{\rm II} | \langle \Psi_{\rm I} | \mathbf{r} | \Psi_{\rm II} \rangle |^2
$$

\n
$$
\frac{1}{2} \sum_{i=1}^{n} \sum_{i=1}^{m} | \langle \mathbf{r} \mathbf{r} | \mathbf{r} | \mathbf{r} \rangle |^2
$$
\n(2)

 $= \frac{1}{n}\sum_{i=1}^n\sum_{j=1}^n |\langle \Psi_{\text{I}i} | \mathbf{r} | \Psi_{\text{II}j} \rangle|^2$ with \bar{v} = mean wave number of the absorption band (cm⁻¹),

 $P =$ dipole strength (cm²),

- $\Psi_{\text{I}} =$ wave function of the ground state, n-fold degenerate,
- Ψ_{II} = wave function of the excited state, m-fold degenerate,
- $r =$ position vector, pointing from the origin to the point of integration. It makes no difference for the calculated value of f which point is chosen as origin. We shall take the origin of the main coordinate system, located on the central ion.

To evaluate the transition probabilities in CuCl²⁻ we have to calculate $n \times m$ integrals $\langle \Psi_I | r | \Psi_{II} \rangle$ for each allowed transition. Ψ_I and Ψ_{II} are wave functions of the whole complex and they may be written as Slater determinantal wave functions, composed from the one electron wave functions of the filled molecular orbitals.

We shall now introduce the following approximations:

a) We assume the CuCl^{2–}-complex to be perfectly tetrahedral. (The real form of the complex determined by X-ray analysis *[13]* is a tetragonally distorted tetrahedron; the Cu-Cl distance is equal to $2.22~\text{\AA}$). It can be shown that ignoring the distortion does not affect the f-values much.

b) We suppose that the one electron molecular orbitals are the same before and after the transition. These molecular orbitals are known from the M.0. calculation described in the preceding paper by P. Ros and G. C. A. SCHUIT. It appears from this calculation that this second assumption is justified.

In the ground state Ψ_I of CuCl²⁻ we have an electron-hole in, say, the molecular orbital ψ_I . In the excited state Ψ_{II} we have a hole in ψ_{II} . We can now prove, making use of the orthonormality relations of the molecular orbitals, that $\langle \Psi_{\rm I} | \mathbf{r} | \Psi_{\rm II} \rangle = \langle \psi_{\rm II} | \mathbf{r} | \psi_{\rm I} \rangle = \langle \psi_{\rm I} | \mathbf{r} | \psi_{\rm II} \rangle$. The determinantal wave functions can simply be replaced by the one electron molecular orbitals containing the hole. Another consequence of the assumptions is that we can use the molecular orbitals calculated for the ground state of tetrahedral CuCl $^{2-}$. The transitions that are allowed and the component integrals of each "degenerate" transition that are not equal to zero are found by applying the multiplication rules of the group T_a [8, 12]. They are listed in Tab. 1. In the last column of this table the relation between the different components of the same transition is indicated. The integrals $\langle \psi_I | r | \psi_{II} \rangle$ that are not equal to zero because of symmetry have to be calculated. Since a molecular orbital ψ is of the form

$$
\psi = C_1 \varphi_M + C_2 \chi_L \text{ with } \varphi_M = \text{atomic orbital of the central ion } M
$$

and $\chi_L = \sum_{\alpha} C_{\alpha} \varphi_{\alpha}; \varphi_{\alpha} = \text{atomic orbital of ligand } L_{\alpha}$

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Allowed transitions, ground state T_{2}	Allowed components of these transitions with polarisation	Factor of a_1 in the direct prod- uct of the basis vectors. This factor determines the relation between the components of a transition
$T_2 \rightarrow A_1$	$\xi \xrightarrow{x} a_1$ $\eta \stackrel{y}{\longrightarrow} a_1$ $\zeta \stackrel{z}{\longrightarrow} a_1$	$\xi x a_1 = \frac{1}{\sqrt{3}} a_1 + \ldots$ $\eta \, y \, a_1 = \frac{1}{\sqrt{3}} a_1 + \ldots$ $\zeta z a_1 = \frac{1}{\sqrt{3}} a_1 + \ldots$
$T_{2} \rightarrow E$	$\xi \stackrel{x}{\longrightarrow} \theta$ $\eta \xrightarrow{y} \theta$	$\xi x \theta = -\frac{1}{2\sqrt{3}}a_1 + \ldots$ $\eta y \theta = -\frac{1}{2\sqrt{3}}a_1 + \ldots$
	$\xi \stackrel{z}{\longrightarrow} \theta$ $\xi \stackrel{\circ}{\longrightarrow} \varepsilon$	$\zeta z \theta = \frac{1}{\sqrt{3}} a_1 + \ldots$ $\xi x \varepsilon = \frac{1}{2} a_1 + \ldots$
	$\eta \stackrel{y}{\longrightarrow} \varepsilon$	$\eta \, y \, \varepsilon \;\; = \;\; - \; \frac{1}{2} \, a_1 \, + \; \ldots$
$T_2 \rightarrow T_1$	$\xi \xrightarrow{y} \gamma$ $\xi \stackrel{z}{\longrightarrow} \beta$	$\xi y \gamma = \frac{1}{\sqrt{6}} a_1 + \ldots$ $\xi z \beta = - \frac{1}{\sqrt{6}} a_1 + \ldots$
	$\eta \stackrel{\ast}{\longrightarrow} \gamma$	$\eta x \gamma = -\frac{1}{\sqrt{6}}a_1 + \ldots$
	$\eta \xrightarrow{z} \alpha$ $\xi \stackrel{x}{\longrightarrow} \beta$	$\eta z \propto - \frac{1}{\sqrt{6}} a_1 + \ldots$ $\zeta x \beta = \frac{1}{\sqrt{6}} a_1 + \ldots$
	$\xi \stackrel{y}{\longrightarrow} \alpha$	$\zeta y \alpha = -\frac{1}{\sqrt{6}}a_1 + \ldots$
$a\,T_{\,2} \rightarrow b\,T_{\,2}$	$\xi \stackrel{y}{\longrightarrow} \zeta$ $\xi \stackrel{z}{\longrightarrow} \eta$	$\xi y \zeta = - \frac{1}{\sqrt{6}} a_1 + \ldots$ $\xi z \eta = -\frac{1}{\sqrt{6}}a_1 + \ldots$
	$\eta \stackrel{x}{\longrightarrow} \zeta$	$\eta x \zeta = -\frac{1}{\sqrt{6}}a_1 + \ldots$ $\eta z \xi = - \frac{1}{\sqrt{6}} a_1 + \ldots$
	$\eta \stackrel{z}{\longrightarrow} \xi$ $\xi \xrightarrow{x} \eta$	$\zeta x \eta = -\frac{1}{\sqrt{6}} a_1 + \ldots$
	$\xi \stackrel{y}{\longrightarrow} \xi$	$\zeta y \xi = -\frac{1}{\sqrt{6}} a_1 + \ldots$

Table 1. *The dipole vector r corresponds to the representation* T_2

we obtain three types of integrals :

$$
a: \langle \varphi_M | r | \varphi_M' \rangle ,b: \langle \varphi_M | r | \chi_L \rangle ,c: \langle \chi_L | r | \chi_L' \rangle .
$$

a: $\langle \varphi_M | r | \varphi_M' \rangle$ is an integral of functions that belong to the same nucleus. With the assumed atomic orbitals, the radial part of which is given by a linear

Fig. 1. Coordinate transformation to calculate the integrals $\langle \varphi_{\alpha} | \mathbf{r} | \varphi_{\beta} \rangle$

combination of Slater functions, an analytical form for these integrals is easily derived. Then a computer-program can be written.

b: $\langle \varphi_M | \mathbf{r} | \chi_L \rangle$ with $\chi_L = \sum C_\alpha \varphi_\alpha$ *r* summation over the ligands L_{α} . The sub- $\text{stitution of } \chi_L \text{ yields:} \sum C_{\alpha} \left\langle \varphi_M \mid \bm{r} \mid \varphi_{\alpha} \right\rangle.$ Now we have integrals of functions that belong to two different nuclei, namely M and L_{α} . These integrals can be evaluated after a rotation of the main coordinate system and a corresponding transformation operating on φ_M and r. By a method given by BALLHAUSEN $[5]$ using elliptical

coordinates [18] we can expand them in A_n and B_n integrals that can be computed. $c: \langle \chi_L | \mathbf{r} | \chi_L \rangle$ may be written as $\sum \sum C_\alpha C_\beta \langle \varphi_\alpha | \mathbf{r} | \varphi_\beta \rangle$. Most of these integrals contain functions that belong to three different nuclei: M , L_{α} and L_{β} . We see

in Fig. 1 that r can be written as $r = r_0 + r'$, r' belonging to L_{α} or L_{β} . This gives: $\langle \varphi_{\alpha} | \mathbf{r} | \varphi'_{\beta} \rangle = \mathbf{r}_{0} \langle \varphi_{\alpha} | \varphi'_{\beta} \rangle + \langle \varphi_{\alpha} | \mathbf{r}' | \varphi'_{\beta} \rangle.$

After rotations of the coordinate systems on the ligands
$$
L_{\alpha}
$$
 and L_{β} we are able to calculate $\langle \varphi_{\alpha} | \varphi'_{\beta} \rangle$ and $\langle \varphi_{\alpha} | \mathbf{r}' | \varphi'_{\beta} \rangle$ by a similar method as used in b.

The computer used is the IBM 1620.

All integrals have been computed for different charge distributions and have been obtained as functions of the charge distribution (A, B, C and D, see M.O.

Electron transition	$v_{\text{max}} \simeq \bar{v}$ $\rm (cm^{-1})$	ε_{\max} $(l \text{ mol}^{-1} \text{ cm}^{-1})$	Exp. t	Theor. f
crystal field $2e \rightarrow 4t_2$	6.000 8,500	shoulder 122	0.0037	0.0075
first charge transfer $t_1 \longrightarrow 4 t_2$	24,500	2400	0.040	0.080
second charge transfer $3t_{0} \longrightarrow 4t_{0}$	34,000	5700	0.120	0.184
third charge transfer $1 e \longrightarrow 4 t_2$ $2 t_2 \longrightarrow 4 t_2$ $2a_1 \longrightarrow 4t_2$	41,000	\simeq 1800	0.032	0.052 0.234 0.316

Table 2

calculation) by means of interpolation formulas. Substitution of the numerical values of A, B, C, D and of the coefficients of the atomic orbitals as found for the ground state of tetrahedral CuCl²⁻ produces the integrals $\langle \psi_{\text{I}} | \mathbf{r} | \psi_{\text{II}} \rangle$ and the oscillator strengths f . The results are given in Tab. 2. The assumed sequence of the bands is discussed later.

Fig. 3. Absorption spectrum of CuCl₄⁻ dissolved in KCl-discs. The part of the spectrum $v > 25,000$ cm⁻¹ is disturbed by scattering of the UV radiation due to the opaqueness of the KCl discs. Measuring in KBr discs was impossible because of the C]--Br- exchange

Fig. 4. Absorption spectrum of CuCl₄⁻ dissolved in acetonitril (cone. 0.001 mol/l, Cl⁻ cone. larger than 0.1 mol/ obtained by dissolving $(C_2 H_5)_4$ NCl \cdot x H_2O and drying over CaCl₂ and P₂O₅)

2. Experimental determination of f: In agreement with the results obtained by several investigators [6, *10, 11, 16]* we found that it makes no difference to the spectrum of $CuCl₄²⁻$ whether it is measured in solid state or in solution. The complex seems to have the same structure independent of the surroundings and of the type of cation. Measuring in solution it is necessary to maintain a large Cl⁻-concentration to prevent the formation of complexes as $CuCl_{4-x}$ Solvent_x [11]. A Zeiss spectrophotometer PM Q II with monoehromator M 4 Q II was used. The obtained spectra are reproduced in Fig. 2, 3, and 4. From the spectrum of Fig. 4 the f-values of the bands have been determined with the formula

$$
f = 4.32 \times 10^{-12} \int_{band} \varepsilon_v dv
$$
 (3)

$$
v = \text{wave number (cm}^{-1}),
$$

$$
\varepsilon_v = \text{molar extinction at wave number } v \text{ (cm}^2 \text{ mol}^{-1}).
$$

The results are given in Tab. 2.

Discussion

I. The largest contributions to the oscillator strengths of all transitions are given by certain integrals $\langle \chi_L | r | \chi_L' \rangle$ (Tab. 3). The integrals that do not occur in this table are at least a factor l0 smaller than the tabulated integrals. A fairly

good approximation to the f -values in the CuCl^{2–}-complex can be given by taking only these $\langle \chi(z) | r | \chi'(z) \rangle$ and $\langle \chi(x, y) | r | \chi'(x, y) \rangle$ integrals into account. This is so because in the molecular orbitals used the coefficients of the Cu 4s- and 4pfunctions and of the Cl χ (3s)-functions are small (see Tab. 4). The $\langle \chi(z)|r|\chi'(z)\rangle$ and $\langle \chi(x, y) | r | \chi'(x, y) \rangle$ integrals can in their turn be approximated by G r_0 . $r_0=x_0, y_0$ or $z_0=\frac{R_{\text{Cu-Cl}}}{\sqrt{2}}$, the projection of the Cu-Cl distance on an axis of the main coordinate system.

G is a factor 1, $\frac{1}{2}$ or $\frac{1}{2}$ $\sqrt{3}$ depending on the coefficients C_{α} in

$$
\chi_L = \sum_{\alpha} C_{\alpha} \varphi_{\alpha}
$$

The last approximation implies in fact neglecting the overlap between the different ligands.

In this way it is possible to calculate the f -values in a fairly good approximation, using only the M.O. coefficients of the ligand orbitals and the distance between the central ionand the ligands (see Tab. 3). This approximation was already used by WOLFSBERG and HELMHOLZ $[20]$. For the CuCl²⁻-complex it appears to be a good one. For other complexes it may also be good when certain conditions are fulfilled.

The approximation of BALLHAUSEN and LIEHR $[2]$, who neglected all terms involving $\langle \chi_L | r | \chi_L' \rangle$ appears to be less satisfactory, for from the exact calculation it follows that these terms give the most important contributions to the transition probabilities.

Table 4

Several investigators e.g. [7, 9, 15] calculated the terms $\langle \varphi_M | \mathbf{r} | \varphi'_M \rangle$ and $\langle \varphi_M | r | \chi_L \rangle$ exactly but approximated the integrals $\langle \chi_L | r | \chi_L' \rangle$ by the method of WOLFSBERG and HELMHOLZ; however since in this approximation terms that have the same order of magnitude as the terms involving $\langle \varphi_M | r | \varphi'_M \rangle$ and $\langle \varphi_M \, | \, r \, | \, \chi_L \rangle$ are neglected it does not make much sense to calculate these integrals exactly.

2. To obtain the positions of the optical bands we must take the difference between the total energies of the excited and ground states. In the M.0. description we saw, however, that it is too difficult to calculate the total energies from the one electron energies and the electron repulsions. Therefore the best we can do is to estimate the positions of the absorption bands from the one electron energies of the ground state. A qualitative scheme for this is given in Fig. 5 for the tetrahedral (T_d) and for the real, distorted tetrahedral, (D_{2d}) form of the $CuCl₄²$ -complex. The distortion causes a splitting of the degenerate energy levels:

$$
\begin{array}{ll}\nT_d & D_{2d} \\
A_1 \longrightarrow A_1 \\
E & \longrightarrow A_1 + B_2 \\
T_1 \longrightarrow E + A_2 \\
T_2 \longrightarrow E + B_2.\n\end{array}
$$

In T_d -symmetry the ground state has a symmetry $T₂$, in D_{2d} a symmetry $B₂$. The allowed transitions are (see Tab. 1):

Ta T 2 l > A~ corresponding to T 2 > E ,, ,, T 2 > T 1 ,, ,, *aT~ > bT 2 ,, ,, D2d B2---+aA 1 B~ > bA 1 B 2 > aE B~ > bE*

(The prefixes a and b are added to distinguish states with the same symmetry.)

the second by:

 $3 t_2 \longrightarrow 4 t_2$ and the third by: in T_d $1e \longrightarrow 4t_2$ or by: $2 t_2 \longrightarrow 4 t_2$ or less likely by: $2 a_1 \longrightarrow 4 t_2$

in T_d

The observed bands in the spectrum are :

a) The crystal field band that has a maximum extinction at 8.500 cm^{-1} and a small shoulder at $6,000 \text{ cm}^{-1}$.

In *Ta* symmetry the only crystal field transition is $T_2 \longrightarrow E$; in terms of one electron transitions: $2e \longrightarrow 4t_{2}$.

In D_{2d} three crystal field transitions are possible :

of which the last one is symmetry-forbidden.

The transition 4 $a_1 \longrightarrow 4 b_2$ corresponds to the large band. The shoulder is caused by the transition $5 e \longrightarrow 4 b_2$ (5e) and $4 b_2$ are the states resulting from the split 4 t_2 state) or by the forbidden transition 2 $b_1 \longrightarrow 4 b_2$.

b) Three charge transfer bands with maxima at 24,500, 34,000 and 41,000 cm⁻¹. We cannot predict the positions of these bands quantitatively from the one electron energies, we can only say something about the sequence in which they appear. So we expect the first charge transfer band to be caused by the transition :

> in D_{2d} $4e \rightarrow 4b$ in D_{2d} $3e \rightarrow 4b$. in D_{2d} $3a_1 \longrightarrow 4b_2$ $2e \rightarrow 4b_2$ $2a_1 \longrightarrow 4b_2$.

This sequence of the absorption bands is affirmed by the comparison of the calculated oscillator strengths with the observed values (see Tab. 2). The third charge transfer band can now be ascribed to the transition $1 e \longrightarrow 4 t_2$ since the other two possibilities require much higher f -values.

Conclusions

1. For all transitions the calculated f -values are about a factor 2 too high with respect to the experimental values. This may partly be ascribed to the fact that eq. (3) is only valid for ideal gases, whereas our absorption spectrum has been measured in solution.

2. The agreement of the calculated and observed relative f-values is still better.

3. The above agreement is good enough to distinguish different transitions from their *f*-values.

4. The transition probabilities, also that of the crystal field transition, are mainly determined by the amount of mixing of the ligand functions in the molecular orbitals. The mixing of the copper $4p$ -function affects the intensities of the bands hardly at all.

5. The agreement of the calculated f -values with the observed values is still better if one assumes that the mixing of the ligand functions in the molecular orbitals is somewhat less.

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